Introduction and application of the

Bezgin-Kolukırık equation to estimate dynamic impact forces on railway tracks due to wheel flats

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ABSTRACT

Wheel flats generate dynamic impact forces on railway tracks and cause rail and track damage. Due to the indeterminate nature of the mechanical interaction of the wheel forces with the railway tracks, estimation of the effects of wheel flats on railway tracks require direct track response measurements from the track or advanced finite element modeling of the wheel and the track contact.

This paper provides a brief introduction to and an application of the Bezgin-Kolukırık equation developed to estimate dynamic impact forces on railway tracks due to wheel flats. This equation is the first analytical equation that can explicitly estimate the dynamic impact forces that arise due to wheel flats on any track through a simple manual calculation.

The equation is based on a method introduced by the first author, (Niyazi Özgür Bezgin) as the Bezgin Method and a concept he introduced as the impact reduction factor. Application of the impact reduction concept to circular motion of the wheel with the second author, (Cengiz Kolukırık) led to the development of the proposed equation.

The paper begins by providing a brief introduction of the equation followed by its application to different railway tracks with varying track stiffness including the primary and secondary spring stiffness elements of the rolling stock and the Hertzian contact stiffness at the wheel-rail interface.

INTRODUCTION

Wheel flats of moving trains wheels generate significant forces on the rail that are many times higher than the static force imposed on the track by the wheel. Wheel flats are flat regions along the circumference of a train wheel formed by locked wheels of a moving train due to aggressive braking or low track adhesion [1].

The mechanical energy generated by the sliding wheel over the rail generates heat that affects the metallurgy of the wheel and the rail near the contact interface and affects the mechanical properties of both the wheel and the rail at and near the wheel-rail contact regions [2].

Plastification and wear of the wheel steel at the sliding contact point results in the formation of the wheel flat. As the wheel rolls along the rail, the wheel flat temporarily eliminates the wheel and rail contact continuity that re-establishes following the wheel flat. The contact force between the rail and the wheel increases beyond the static wheel force value as the wheel flat hits the rail and the estimation of this force is not an easy matter. The matter is complicated due to the highly indeterminate nature of the wheel and rail contact as well as the indeterminate response mechanism of the track to imposed forces.

The Winkler's analytical representation of a vertically deformed railway track under vertical wheel forces provides an accepted model to estimate the response of a railway track to imposed vertical wheel forces.

However, an analytical representation of the dynamic impact forces due to wheel flat was not available until the introduction of the Bezgin-Kolukırık Equation in January 2019 at the 98th Transportation Research Board Meeting Conference.

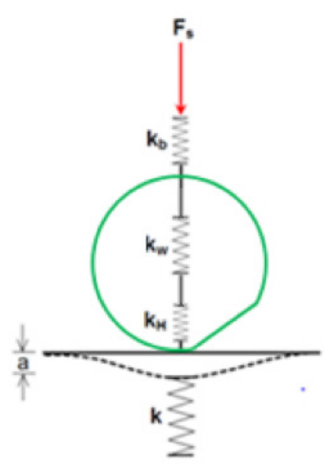

Figure 1: Representation of the bogie-wheel-track system stiffness with a depiction of a wheel with a flat

This paper presents a summary of the proposed equation, the details for which one can find at the publicly released resource where preliminary verifications of the Bezgin-Kolukırık Equation with two numerical studies by other authors [3].

This paper applies the Bezgin-Kolukırık Equation for increasing wheel flat lengths up to 50 mm and for selected track and rolling stock stiffness values for train speeds up to 300 km/h. The next section and the section that follows introduce and apply the Bezgin-Kolukırık Equation and end with a discussion and planned future studies.

BEZGIN-KOLUKIRIK EQUATION

Figure 1 depicts a bogie-wheel-track system that includes the secondary and primary spring stiffness values of the bogie and the wheel k_b and k_w along with the Hertzian contact stiffness of the wheel with the rail k_H and the track stiffness per rail "k". The deflection of the track where the wheel contacts with the track is "a". The total deflection of the system including the primary and secondary springs and the Hertzian contact is a'. The figure depicts the wheel with a flat. Fs represent the static force exerted on the system.

Equation 1 shows the equivalent stiffness of the system keq in terms of the individual stiffness values of its constituent elements.

Figure 2 is a close-up sketch of the wheel and rail contact, providing a geometrical description of the wheel flat with respect to the wheel geometry. The sketch does not present local deformations at point A and the Hertzian contact conditions. The static gravitational vertical force due to the tributary mass of the wheel exerted on the track at point A exerts at point B as the wheel rolls along the track towards right.

Depending on the translational speed of the train and the temporal angular speed of the wheel around point A that arises, the force exerted on the track due to the tributary mass of the wheel increases beyond it static value at point B.

If the wheel rolls along very slowly at a speed almost equal to zero, the tributary mass of the wheel gently releases back on the track at point B producing a force on the track that is almost equal to its static value at point A.

However, if the translational speed of the train is high, depending on the wheel radius and the wheel flat length, the tributary mass of the wheel no longer places on the track gently at point B but smashes on to the track rapidly producing a force on the track far beyond it static value.

One may consider this as a mass "m" being placed gently on a supporting surface from a height "h" or dropped on to the surface at a free-fall speed from height "h" or thrown at the surface at a speed greater than the free-fall speed from a height of "h".

The force exerted on the supporting surface by the mass "m", will become consecutively higher. Based on this analogy, Bezgin developed a method based on the law of conservation of energy, rules of kinematics and a new concept of impact reduction factor (f) that he introduced as shown in **Equation 2** [4].

The method considers the tributary mass carried by the wheel including the mass of the wheel. The new concept of impact reduction factor; originally developed for a descending track profile, correlates the hypothetical time to free-fall from a height "h", which is the variation in track profile, to the time the train requires passing along the rough length of track (L) along which this profile change develops as presented in **figure 3** [4].

Depending on the translational speed of the train, the wheel depicted at location-1 in **figure 3** can generate a higher force and thus a higher deflection (c) on the track at location-2.

Application of the Bezgin Method to a descending track profile resulted in Equation 3, which is the "Bezgin Equation for descending track profile", that provides an estimate of the dynamic impact force factor with respect to the static force of the wheel represented initially by K_B and then renamed as $K_{B,d}$ where "d" represents the descending track condition [4,5]. The only stiffness component considered in K_B is the track stiffness.

Consideration of the system stiffness keq and a factor for damping "s" resulted in **Equation 4**, which is the "Extended Bezgin Equation for descending track condition".

Previous studies show that estimates of $K_{B,d}$ compares well with some of the existing empirical equations [4,6]. Recent studies show that the estimates of $K'_{B,d}$ compare well with estimates of high-end finite element analyses with a deviation of less than 10% (7).

Bezgin Method is an energy based method and has applicability to any track or wheel roughness conditions that generate dynamic impact forces on railway tracks. Such being the case, the authors extended the concept of impact reduction factor originally devised for translational speed of wheels to angular speed of wheels and obtained **Equation 5**. This equation is geometrically developed based on **figure 2**, details for which can be found elsewhere [3,8].

Use of **Equation 5** in **Equation 4** resulted in **Equation 6**, which is the Extended Bezgin-Kolukırık equation that estimates dynamic impact forces due to wheel flats [3].

The authors second work together provided a better geometric presentation and evaluation of the wheel-rail interaction compared to the first [3,8]. K'_{B3} equation considers the system stiffness and the resultant system deflection a' and excludes system damping "s" since the duration of impact force due to wheel flat is extremely short.

This is the fifth of the equations developed by the Bezgin Method to assess the dynamic impact forces due to various track and wheel roughness. If one considers the stiffness of the track only, the track deflection "a" is used in **Equation 6** instead of the system deflection a', and K'_{B3} becomes K_{B3}.

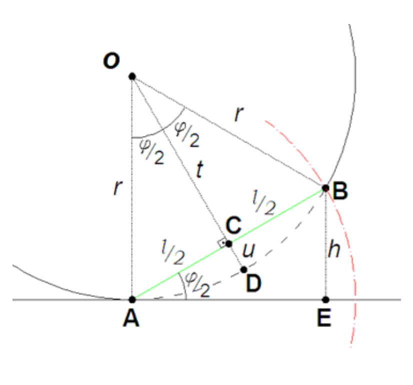

Figure 2: Geometric depiction of a wheel flat [3]

$$\frac{1}{k_{eq}} = \frac{1}{k_b} + \frac{1}{k_w} + \frac{1}{k_H} + \frac{1}{k}$$

Equation 1

$$f = 1 - \frac{t_{fall}}{t_{pass}}$$

Equation 2

$$K_{B,d} = 1 + \sqrt{\frac{2h}{a}(1 - f)}$$

Equation 3

$$K'_{B,d} = 1 + \sqrt{\frac{2h}{a'}(1 - f - s)}$$

Equation 4

$$f = 1 - \frac{t_{fall}}{t_{spin}} = 1 - \frac{2.\sin\frac{\varphi}{2}\sqrt{\frac{r}{g}}}{\varphi.r/_{2.v}} = 1 - \frac{4.v.\sin\frac{\varphi}{2}}{\varphi.\sqrt{r.g}}$$

Equation 5

$$K'_{B3} = 1 + 4.\sin\frac{\varphi}{2}\sqrt{\frac{\sqrt{r}.v.\sin\frac{\varphi}{2}}{a'.\varphi.\sqrt{g}}}$$

Equation 6

D (mm)	*l* (mm)	ϕ (rad)	ϕ (deg)	Train speeds (km/h)											
				50		100		150		200		250		300	
				K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}
780	5	0.013	0.7	**1.2**	***1.6***	1.2	1.8	1.3	2.0	1.3	2.1	1.3	2.2	**1.4**	***2.3***
	10	0.026	1.5	**1.3**	***2.1***	1.4	2.6	1.5	2.9	1.6	3.2	1.7	3.5	**1.7**	***3.7***
	15	0.038	2.2	**1.5**	***2.7***	1.6	3.3	1.8	3.9	1.9	4.3	2.0	4.7	**2.1**	***5.0***
	20	0.051	2.9	**1.6**	***3.2***	1.9	4.1	2.1	4.8	2.2	5.4	2.4	5.9	**2.5**	***6.4***
	25	0.064	3.7	**1.8**	***3.8***	2.1	4.9	2.3	5.8	2.5	6.5	2.7	7.2	**2.9**	***7.7***
	30	0.077	4.4	**1.9**	***4.3***	2.3	5.7	2.6	6.7	2.8	7.6	3.0	8.4	**3.2**	***9.1***
	35	0.090	5.1	**2.1**	***4.9***	2.5	6.5	2.8	7.7	3.1	8.7	3.4	9.6	**3.6**	***10.4***
	40	0.103	5.9	**2.2**	***5.4***	2.7	7.2	3.1	8.6	3.4	9.8	3.7	10.9	**4.0**	***11.8***
	45	0.115	6.6	**2.4**	***6.0***	2.9	8.0	3.4	9.6	3.7	10.9	4.1	12.1	**4.4**	***13.1***
	50	0.128	7.4	**2.5**	***6.5***	3.2	8.8	3.6	10.5	4.0	12.0	4.4	13.3	**4.7**	***14.5***
840	5	0.012	0.7	**1.1**	***1.5***	1.2	1.7	1.2	1.9	1.3	2.0	1.3	2.1	**1.3**	***2.2***
	10	0.023	1.3	**1.3**	***2.0***	1.4	2.4	1.5	2.8	1.6	3.0	1.6	3.3	**1.7**	***3.5***
	15	0.035	2.0	**1.4**	***2.5***	1.6	3.2	1.7	3.6	1.8	4.1	1.9	4.4	**2.0**	***4.7***
	20	0.047	2.7	**1.6**	***3.0***	1.8	3.9	2.0	4.5	2.1	5.1	2.3	5.6	**2.4**	***6.0***
	25	0.058	3.3	**1.7**	***3.5***	2.0	4.6	2.2	5.4	2.4	6.1	2.6	6.7	**2.7**	***7.2***
	30	0.070	4.0	**1.8**	***4.1***	2.2	5.3	2.5	6.3	2.7	7.1	2.9	7.8	**3.1**	***8.5***
	35	0.081	4.7	**2.0**	***4.6***	2.4	6.0	2.7	7.2	3.0	8.1	3.2	9.0	**3.4**	***9.7***
	40	0.093	5.3	**2.1**	***5.1***	2.6	6.8	3.0	8.1	3.3	9.1	3.5	10.1	**3.8**	***11.0***
	45	0.105	6.0	**2.3**	***5.6***	2.8	7.5	3.2	8.9	3.5	10.2	3.8	11.2	**4.1**	***12.2***
	50	0.116	6.7	**2.4**	***6.1***	3.0	8.2	3.4	9.8	3.8	11.2	4.1	12.4	**4.4**	***13.5***
920	5	0.011	0.6	**1.1**	***1.5***	1.2	1.7	1.2	1.8	1.3	2.0	1.3	2.1	**1.3**	***2.2***
	10	0.022	1.2	**1.3**	***2.0***	1.4	2.4	1.5	2.7	1.5	2.9	1.6	3.2	**1.7**	***3.4***
	15	0.033	1.9	**1.4**	***2.5***	1.6	3.1	1.7	3.5	1.8	3.9	1.9	4.3	**2.0**	***4.6***
	20	0.043	2.5	**1.5**	***2.9***	1.8	3.8	1.9	4.4	2.1	4.9	2.2	5.4	**2.3**	***5.8***
	25	0.054	3.1	**1.7**	***3.4***	2.0	4.4	2.2	5.2	2.3	5.9	2.5	6.4	**2.6**	***7.0***
	30	0.065	3.7	**1.8**	***3.9***	2.1	5.1	2.4	6.1	2.6	6.8	2.8	7.5	**3.0**	***8.2***
	35	0.076	4.4	**1.9**	***4.4***	2.3	5.8	2.6	6.9	2.9	7.8	3.1	8.6	**3.3**	***9.3***
	40	0.087	5.0	**2.1**	***4.9***	2.5	6.5	2.9	7.7	3.2	8.8	3.4	9.7	**3.6**	***10.5***
	45	0.098	5.6	**2.2**	***5.4***	2.7	7.2	3.1	8.6	3.4	9.8	3.7	10.8	**4.0**	***11.7***
	50	0.109	6.2	**2.3**	***5.9***	2.9	7.9	3.3	9.4	3.7	10.7	4.0	11.9	**4.3**	***12.9***

Table 1: Estimation of K_{B3} and K'_{B3} for k=30 kN/mm

D (mm)	*l* (mm)	ϕ (rad)	ϕ (deg)	Train speeds (km/h)											
				50		100		150		200		250		300	
				K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}	K'_{B3}	K_{B3}
780	5	0.013	0.7	**1.2**	***1.8***	1.2	2.1	1.3	2.3	1.3	2.6	1.3	2.7	**1.4**	***2.9***
	10	0.026	1.5	**1.3**	***2.6***	1.4	3.2	1.5	3.7	1.6	4.1	1.7	4.5	**1.8**	***4.8***
	15	0.038	2.2	**1.5**	***3.3***	1.7	4.3	1.8	5.0	1.9	5.7	2.0	6.2	**2.1**	***6.7***
	20	0.051	2.9	**1.6**	***4.1***	1.9	5.4	2.1	6.4	2.2	7.2	2.4	8.0	**2.5**	***8.6***
	25	0.064	3.7	**1.8**	***4.9***	2.1	6.5	2.3	7.7	2.6	8.8	2.7	9.7	**2.9**	***10.5***
	30	0.077	4.4	**1.9**	***5.7***	2.3	7.6	2.6	9.1	2.9	10.3	3.1	11.5	**3.3**	***12.4***
	35	0.090	5.1	**2.1**	***6.5***	2.5	8.7	2.9	10.4	3.2	11.9	3.4	13.2	**3.7**	***14.4***
	40	0.103	5.9	**2.2**	***7.2***	2.8	9.8	3.2	11.8	3.5	13.5	3.8	14.9	**4.0**	***16.3***
	45	0.115	6.6	**2.4**	***8.0***	3.0	10.9	3.4	13.1	3.8	15.0	4.1	16.7	**4.4**	***18.2***
	50	0.128	7.4	**2.6**	***8.8***	3.2	12.0	3.7	14.5	4.1	16.6	4.5	18.4	**4.8**	***20.1***
840	5	0.012	0.7	**1.1**	***1.7***	1.2	2.0	1.2	2.2	1.3	2.4	1.3	2.6	**1.4**	***2.8***
	10	0.023	1.3	**1.3**	***2.4***	1.4	3.0	1.5	3.5	1.6	3.9	1.6	4.2	**1.7**	***4.5***
	15	0.035	2.0	**1.4**	***3.2***	1.6	4.1	1.7	4.7	1.9	5.3	2.0	5.8	**2.1**	***6.3***
	20	0.047	2.7	**1.6**	***3.9***	1.8	5.1	2.0	6.0	2.1	6.8	2.3	7.4	**2.4**	***8.1***
	25	0.058	3.3	**1.7**	***4.6***	2.0	6.1	2.2	7.2	2.4	8.2	2.6	9.0	**2.8**	***9.8***
	30	0.070	4.0	**1.9**	***5.3***	2.2	7.1	2.5	8.5	2.7	9.6	2.9	10.7	**3.1**	***11.6***
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	45	0.105	6.0	**2.3**	***7.5***	2.8	10.2	3.2	12.2	3.6	14.0	3.9	15.5	**4.2**	***16.9***
	50	0.116	6.7	**2.4**	***8.2***	3.0	11.2	3.5	13.5	3.9	15.4	4.2	17.1	**4.5**	***18.6***
920	5	0.011	0.6	**1.1**	***1.7***	1.2	2.0	1.2	2.2	1.3	2.4	1.3	2.5	**1.3**	***2.7***
	10	0.022	1.2	**1.3**	***2.4***	1.4	2.9	1.5	3.4	1.5	3.8	1.6	4.1	**1.7**	***4.4***
	15	0.033	1.9	**1.4**	***3.1***	1.6	3.9	1.7	4.6	1.8	5.1	1.9	5.6	**2.0**	***6.1***
	20	0.043	2.5	**1.5**	***3.8***	1.8	4.9	2.0	5.8	2.1	6.5	2.2	7.2	**2.3**	***7.7***
	25	0.054	3.1	**1.7**	***4.4***	2.0	5.9	2.2	7.0	2.4	7.9	2.5	8.7	**2.7**	***9.4***
	30	0.065	3.7	**1.8**	***5.1***	2.2	6.8	2.4	8.2	2.6	9.3	2.8	10.2	**3.0**	***11.1***
	35	0.076	4.4	**2.0**	***5.8***	2.4	7.8	2.7	9.3	2.9	10.6	3.1	11.8	**3.4**	***12.8***
	40	0.087	5.0	**2.1**	***6.5***	2.6	8.8	2.9	10.5	3.2	12.0	3.5	13.3	**3.7**	***14.5***
	45	0.098	5.6	**2.2**	***7.2***	2.7	9.8	3.1	11.7	3.5	13.4	3.8	14.8	**4.0**	***16.2***
	50	0.109	6.2	**2.4**	***7.9***	2.9	10.7	3.4	12.9	3.7	14.8	4.1	16.4	**4.4**	***17.9***

Table 2: Estimation of K_{B3} and K'_{B3} for k=60 kN/mm

APPLICATION OF THE BEZGIN-KOLUKIRIK EQUATION

This section presents an in-depth application of the proposed equation for various values of wheel diameters, wheel flat lengths, train speeds and track stiffness values selected to cover a wide range of possible cases. **Tables 1 and 2** present the K'_{B3} and K_{B3} estimates for track stiffness values per rail of k=30 kN/mm and 60 kN/mm respectively that are aimed to represent soft to normal track bed stiffness values.

The selected representative bogie spring stiffness is k_w=3.6 kN/mm, wheel spring stiffness is k_b=8.1 kN/mm and the Hertzian contact stiffness is k_H=1600 kN/mm [3]. One can clearly see from a comparison of these tables that the dynamic impact effect of a wheel flat inversely relates to wheel diameter and directly relates to train speed, wheel flat length, track stiffness and system stiffness values. One can also observe from the comparison of these tables that the effect of the system stiffness on the developing dynamic impact forces is very high.

Typically, the primary and secondary spring stiffness values of the rolling stock are much lower than the track stiffness resulting in an equivalent system stiffness that is much lower than the stiffness of the track alone. This is clearly visible in the tables such that K_{B3} estimates are about 1.5 to 5 times higher than K'_{B3} estimates depending on the length of the wheel flat, train speed and wheel diameter.

This finding is an important indicator of the necessity of proper functioning of primary and secondary spring elements. Should they become non-functional, the dynamic impact forces acting on the track become extremely high. The estimated dynamic impact force factors have been underlined and highlighted for v=50 km/h and v=300 km/h for clarity.

Figure 4 plots the values presented in **Table 2**. One can clearly observe that the variation of the estimated dynamic impact force factors is linear. This trend has been verified with the results of a numerical study conducted on ANSYS® by other researchers [3]. The estimated values by the proposed equation has been shown to agree very well with one verification study and agree to some degree with another verification study [3]. The referenced study is available online for free access.

Two sets of numbers on each figure are highlighted for low speed and high axle load freight trains and high speed and low axle load passenger trains. The wheel flat limits allowed for passenger trains and for freight trains are taken as l=30 mm and l=50 mm respectively [9,10]. One can reach a conclusion immediately that if a limit on a wheel flat is specified this must relate to the particular wheel diameter.

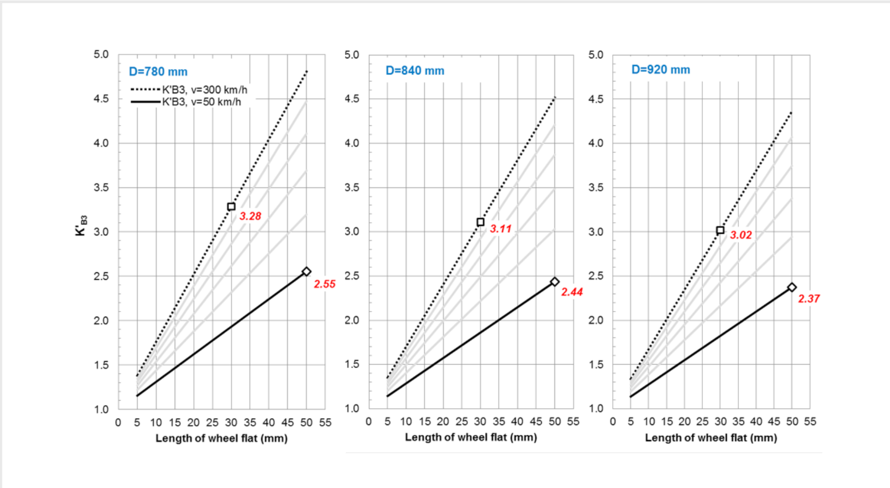

Figure 4: K'_{B3} estimates for track stiffness k=60 kN/mm.

The ratios of the estimated dynamic impact factors for l=50 mm and l=30 mm vary between 1.27 and 1.29. For a practical purpose, this ratio can be stated as 1.3. In other words, the limit for the dynamic impact force factors for trains with wheels with a diameter of D=920 mm are about K'_{B3}=3.1 for passenger trains and K'_{B3}=2.4 for freight trains.

DISCUSSION AND CONCLUSION

The proposed approach and the proposed equation have several practical uses and benefits for proactive track management. The first benefit of the Bezgin-Kolukırık Equation is that it is an analytical equation that requires a couple of basic input parameters to manually yield an estimate of dynamic impact fore factors due to wheel flats. The proposed equation negates the necessity to resort to advanced numerical methods to arrive at an estimate due to wheel flats and empowers the track engineer to come up with an independent estimate and judge the situation before he or she considers or requires to move on to more advanced numerical methods. The proposed equation captures the trend between the generated dynamic impact forces and track and rolling stock parameters. Most importantly, the proposed equation allows a limit on the wheel flat lengths or judge the effectiveness of a wheel flat length.

As a rough estimate of the developing contact stresses due to wheel flats let us consider the following. If the static axle force of a freight train moving at v=50 km/h with a wheel diameter D=840 mm on a track with a stiffness of k=60 kN/mm is 250 kN and the allowed wheel flat length limit is l=50 mm, the estimated dynamic impact wheel force on the track based on **figure 4** would be
F_i=2.44 * 125 kN = 305 kN. If the Hertzian contact area between the wheel and the rail is A=300 mm^2, the contact stress under static conditions is
sigma-static=125 kN/300 mm^2=417 MPa.
The contact stress under dynamic impact conditions would increase to
sigma-impact=2.44 * 417 MPa = 1017 MPa, which is significantly high.If one requires that railhead contact stresses due to wheel flats to be limited to 600 MPa, one must specify a wheel flat limit for the aforementioned conditions such that the produced dynamic impact force factor due to the wheel flat is K'_{B3}=600 MPa/417MPa = 1.43, which develops for a wheel flat length of
l=17 mm for D=840 mm.

One can incorporate other stiffness elements within the track-rolling stock system depicted in **figure 1**, such as under sleeper pads and under rail pads, should they exist. The estimates of the proposed equation can also be used to assess the compressive stress conditions in the ballast layer and the subgrade that develop due to wheel flats and evaluate the conditions of possible ballast crushing under the sleepers. The authors have also developed a way to include the effects of local deformations due to Hertzian contact conditions on the dynamic impact forces due to wheel flats and developed the equation K'_{B3-H} [3]. The presented study currently progresses with studies for further numerical and experimental verification of the proposed Bezgin-Kolukırık Equation.

Bezgin-Kolukırık equation is a part of set of equations developed by the Bezgin Method known as the Extended Bezgin Equations. The other equations involve the analytical estimations of dynamic impact forces due to track stiffness transitions and track profile variations. An application titled ALLTRACK® v1 [11] is also available online that includes the current state of the complete set of equations developed by the Bezgin Method.

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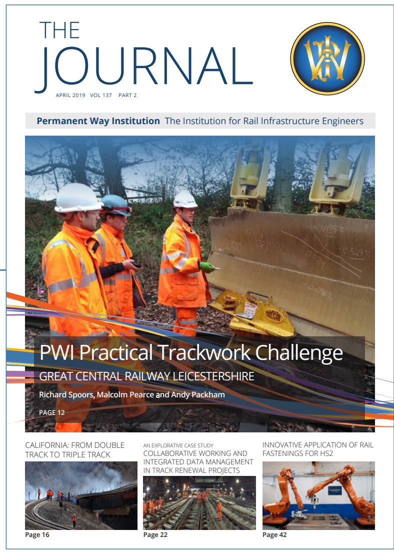

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