BACK TO BASICS ▸ Equilibrium cant

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If a young engineer were to ask an old railwayman like myself how much cant they should apply to a particular curve, he would immediately get inundated with questions.

What's the existing cant, the existing radius?

What is the rail, weight, section and condition?

What's the condition of the ballast and drainage?

What rolling stock currently uses the route?

What are the positions of the nearest signals, S&C, platforms and structures?

And you may be asking yourself why?

Well because these are all variables and they are all factors amongst many others in making an informed engineering decision as to how much cant to apply on that curve, but there is one constant in the process and that is the number 11.82 and it appears in the equilibrium cant equation.

$$Eq = \frac{11.82\,Ve^2}{R}$$

Figure 1 above - Equilibrium cant equation, where Ve is the line speed in kph and R is the radius in metres, and Eq, the equilibrium cant is in mm.

And if there is one equation that needs imprinting on every track engineer's brain it is this one.

We need to start by defining equilibrium cant. See figures 2 & 3.

When a train travels around a curve there is an outward force which when combined with the weight gives a resultant force that no longer acts vertically. Load is transferred towards the outer rail.

If we raise the outer rail (apply cant), there is, theoretically an amount of cant which results in the resultant force acting perpendicular to the sleeper. This is known as the Equilibrium Cant (Eq), the cant at a particular speed at which the resultant force is perpendicular to the running plane of the rails.

So if we use an example of a train travelling at 100 kph around a 900m curve then:

$$Eq = \frac{11.82Ve^2}{R}$$

$$Eq = \frac{11.82 \times 100 \times 100}{900}$$

Eq = 131.3mm

This means that in this example applying 131.3mm of cant to the track would, at a speed of 100kph, result in no lateral forces on the rail. No Maintenance Engineer could maintain a cant of 131.3mm and we therefore apply cant in 5mm blocks. On the UK mainline network we often operate a mixed traffic railway on the same line. That is inter-city trains travelling at high speeds up to 125mph, suburban services with lighter axle weights travelling at

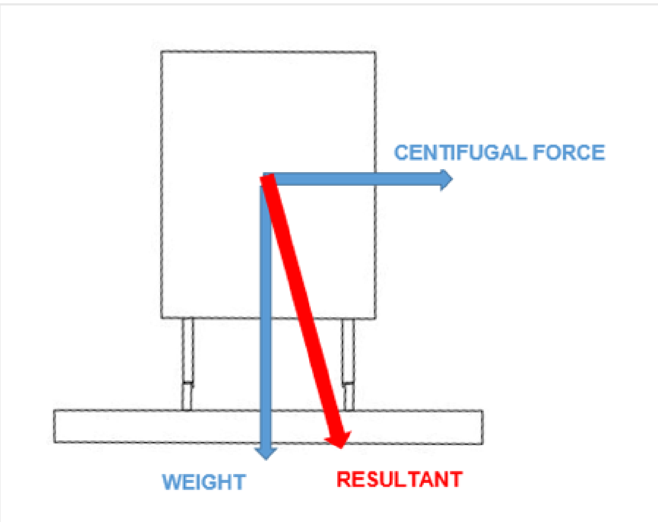

Figure 2: Forces applied by a train to the track travelling around a curve with zero cant.

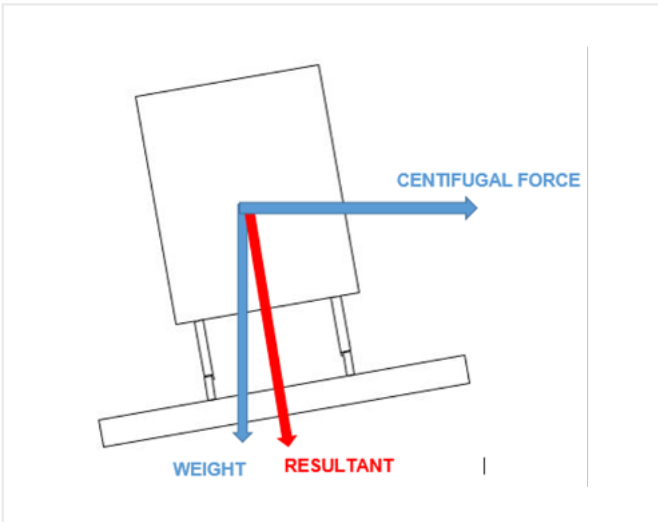

Figure 3: Forces applied by a train to the track after the application of cant.

75mph and also freight trains with much higher axle weights at lower linespeeds. The amount of cant that we apply has to take into account the frequency and types of rolling stock using the route. What is equilibrium cant for the inter-city train would certainly be an excess of cant (over canted) for the freight train in this example. We therefore apply a cant of less than equilibrium cant.

If the applied cant is less than the calculated Equilibrium Cant for the speed under consideration, then the track is 'deficient in cant' (ie it has less than the amount required for Equilibrium) So, the **cant deficiency** = Equilibrium cant minus the Applied Cant. See figure 4.

It used to be taught in railway circles that the amount of cant to be applied was 2/3rds of the Equilibrium Cant value which in this example would be around 90mm. Two thirds of 131 mm is 87mm rounded up to the nearest 5mm step which is 90mm. This would give us a deficiency value of 41mm. The thinking behind applying 2/3rds cant to deficiency was that it was the best compromise for that mixed traffic railway. Too much cant deficiency results in sidewear to the high rail, and an excess of cant in the case of a freight train causes "mushrooming" or compression to the rail head of the low rail, both of these conditions need to be monitored by the track maintenance team.

The advent of modern analysis tools such as TrackEx has meant that we have discovered that cant deficiency can actually be the engineer's friend and as the resultant of forces is slightly towards the high rail, we have found that this helps with the steering of the bogies around the curve, smoothing the passenger ride and that cant deficiency can aid in the reduction of the propagation of rolling contact fatigue. The result of these discoveries is that cant can be applied at a value closer to 50% of the equilibrium value which in this example would be 70mm. But as stated in the very first paragraph there is no automatic formula that will calculate the exact value of cant that you should apply and you need to consider all of the variables.

But what of the constant 11.82? The equilibrium cant formula is based upon the 'balancing' of the outward centrifugal force with the inward force from the train 'sliding' down the angle of cant.

The factor of centrifugal force representing acceleration and which may be assumed to be acting outwards = $\frac{V^2}{R}$ measured in metres per sec. per sec.

The inward acceleration due to cant = $\frac{e_a g}{s}$ measured in metres per sec. per sec

For equilibrium these two accelerations are equal

$$\frac{V^2}{R} = \frac{e_a g}{s} \quad \text{whence } e_a = e = \frac{sV^2}{gR}$$

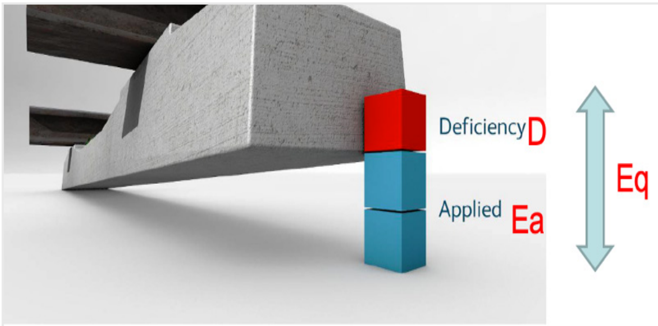

Figure 4: Cant Deficiency = Equilibrium Cant - Applied Cant.

It should be noted that 'S' is NOT the track gauge but is the distance between the centre of the rails and cant is measured between the centres of the rails (think how a cant gauge sits on the rails). For any railway the only two variables that we use are the radius of the curve and the line speed. See figure 5.

It should be obvious that 11.82 is an approximation of the answer rounded up two decimal places. But if you are working on another railway administration such as NIR where the gauge is 1.600m you could calculate using the above formula that the constant to be used for 1600mm gauge with CEN56E rails with a 70mm rail head thickness would be 13.135 .

NR/L2/TRK/2049 states that 11.82 is defined for gauge but should be used as a constant for all standard gauge Network Rail lines regardless of gauge variations. And a standard-gauge railway is defined as a railway with a track gauge of 1.435m or 4ft 8 ½ in. which of course includes TfL, DLR and our major metro tram systems. The only public railway I can think of in the UK which has a different gauge is the 1219mm of Glasgow's Clockwork Orange and I will let you calculate the equilibrium cant constant for that yourselves.

There is a far more detailed explanation of the maths on the excellent Pway blog written by Constantin Ciobanu at https://pwayblog.com/2015/10/29/11-82_cant-deficiency-un-compensated-acceleration-pway/

Figures in this article are courtesy of the PWI Track Engineering Diploma. See page 65 for more details on this Diploma.

This is part 2 of a series of BACK TO BASIC articles.

Part 1, Vertical Alignment Design, can be found in the new online Knowledge Hub.

In the above, s = distance between centres of rails (in m) ; g = acceleration due to gravity (in m/sec) ; r = curve radius in m and v = speed in km/h; e = cant

S = 1.502m if gauge is 1.432m or 1.505m if gauge is 1.435m

G = 9.81m /s/s

Converting km/h to m/s

To ensure consistent units and obtain e in mm:-

Converting m to mm

$$e = \frac{1.502 \times V^2 \times 1000 \times 1000 \times 1000}{9.81 \times R \times 3600 \times 3600}$$

e = 11.8139716 x $\frac{V^2}{R}$

NB if gauge is 1.435m, equation is e = 11.837568 x $\frac{v^2}{R}$

Figure 5: Derivation of the constant in the Equilibrium Cant Equation.

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